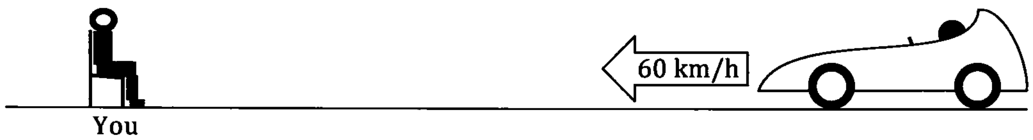
Chapter 6.

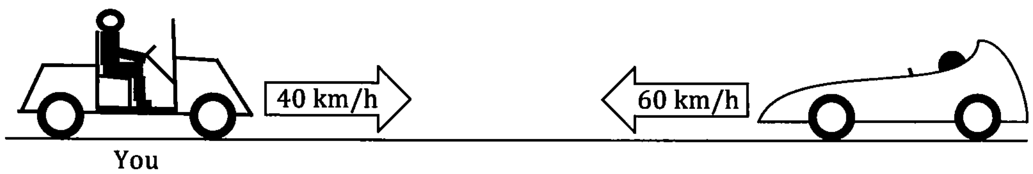
Relative displacement and relative velocity.

Situation

The diagram below shows "you" sitting in a chair watching a sports car approach at 60 km/h. Quite clearly what you see is the sports car approaching at 60 km/h.

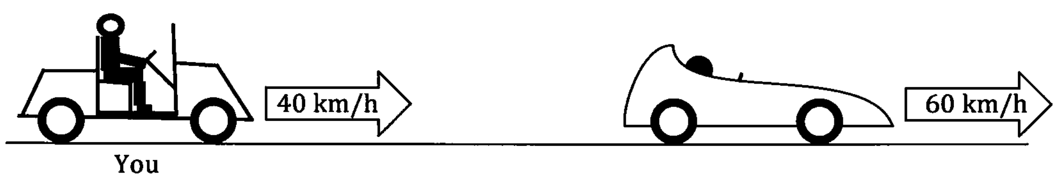


Suppose now that as well as the sports car approaching you at 60 km/h, you are approaching the sports car at 40 km/h. This situation is shown below.



What you would see in this case is the same as you would see if you were not moving and the sports car were approaching you at 100 km/h. We say that the situation **relative to you** is that the sports car is approaching at 100 km/h.

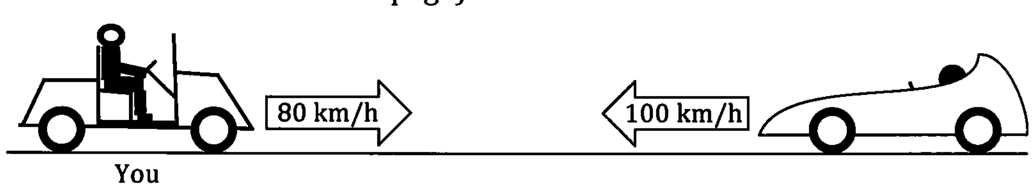
Consider now the situation shown below.



In this case the situation relative to you is that the sports car is moving away from you at 20 km/h.

In each of the following situations describe the motion of the sports car relative to "you". (The situations continue on the next page).

1.



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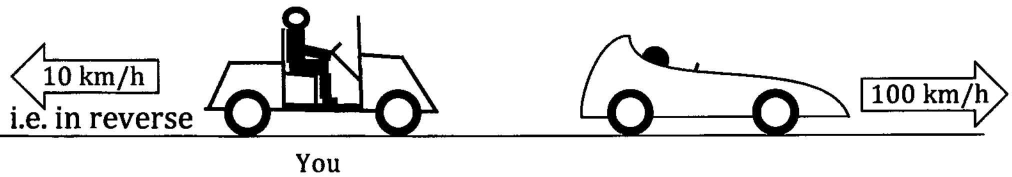

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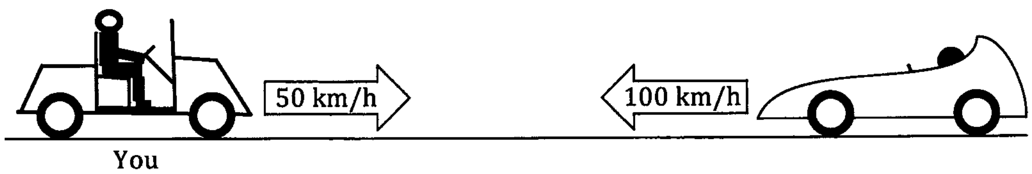

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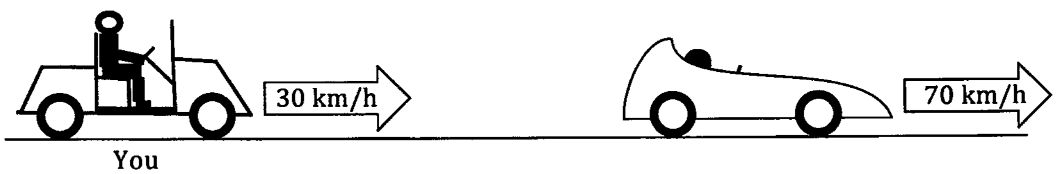

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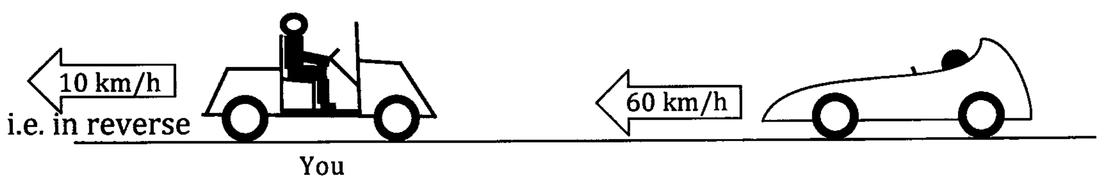

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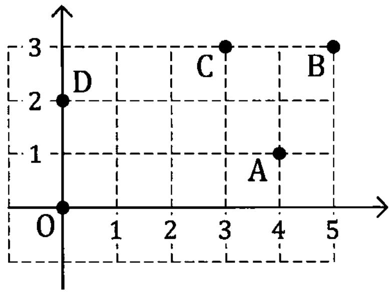


Relative displacement.

Consider the points O, A, B, C and D shown on the right.

With respect to the origin, O, the position vectors of A, B, C and D are:

$$\mathbf{r}_A = \overrightarrow{OA} = 4\mathbf{i} + \mathbf{j},$$

$$\mathbf{r}_B = \overrightarrow{OB} = 5\mathbf{i} + 3\mathbf{j},$$

$$\mathbf{r}_C = \overrightarrow{OC} = 3\mathbf{i} + 3\mathbf{j},$$

$$\mathbf{r}_D = \overrightarrow{OD} = 0\mathbf{i} + 2\mathbf{j}.$$

These vectors give the **displacements** of each point from O.
However, if we were situated at A it may be more useful to know the displacement of each of the other points relative to our own position, A. Writing the position vector of B relative to A (or the displacement of B relative to A) as ${}_B\mathbf{r}_A$ it follows that:

$${}_B\mathbf{r}_A = \overrightarrow{AB} = \mathbf{i} + 2\mathbf{j},$$

$${}_C\mathbf{r}_A = \overrightarrow{AC} = -\mathbf{i} + 2\mathbf{j},$$

$${}_D\mathbf{r}_A = \overrightarrow{AD} = -4\mathbf{i} + \mathbf{j},$$

$${}_O\mathbf{r}_A = \overrightarrow{AO} = -4\mathbf{i} - \mathbf{j}.$$

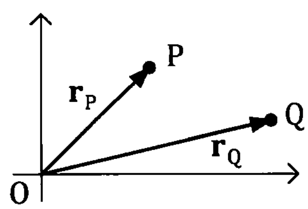
It follows that

$${}_P\mathbf{r}_Q = \overrightarrow{QP}$$

$$= -\mathbf{r}_Q + \mathbf{r}_P$$

i.e.

$$\boxed{{}_P\mathbf{r}_Q = \mathbf{r}_P - \mathbf{r}_Q}$$



Though this result is shown "boxed", in most cases it is better to think about what is being asked for rather than to simply substitute into the rule.

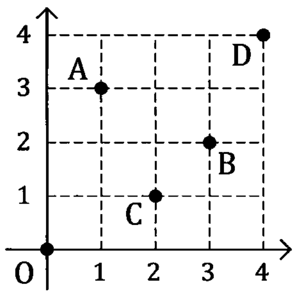
Note: We could write the position vector of A relative to O as ${}_A\mathbf{r}_O$ but, because it is usual to give position vectors relative to the origin, we tend to simply write this as $\mathbf{r}_A$, as we did at the top of this page.

If we are asked for the position vector of a point then this should be taken as being relative to the origin, position vector $0\mathbf{i} + 0\mathbf{j}$, unless stated otherwise.

Example 1

The diagram on the right shows the origin O and the points A, B, C and D. Find each of the following in the form a**i** + b**j**.

(a) The position vector of A,
(b) The position vector of A relative to B,
(c) The displacement of B relative to A.
(d) The displacement of C relative to A.
(e) The position vector of A relative to C.
(f) ${}_D\mathbf{r}_A$ (g) ${}_A\mathbf{r}_D$ (h) ${}_C\mathbf{r}_D$



(a) $\mathbf{r}_A = \mathbf{i} + 3\mathbf{j}$

(b) ${}_A\mathbf{r}_B = \overrightarrow{BA}$
$= -2\mathbf{i} + \mathbf{j}$

(c) ${}_B\mathbf{r}_A = \overrightarrow{AB}$
$= 2\mathbf{i} - \mathbf{j}$

(d) ${}_C\mathbf{r}_A = \overrightarrow{AC}$
$= \mathbf{i} - 2\mathbf{j}$

(e) ${}_A\mathbf{r}_C = \overrightarrow{CA}$
$= -\mathbf{i} + 2\mathbf{j}$

(f) ${}_D\mathbf{r}_A = \overrightarrow{AD}$
$= 3\mathbf{i} + \mathbf{j}$

(g) ${}_A\mathbf{r}_D = \overrightarrow{DA}$
$= -3\mathbf{i} - \mathbf{j}$

(h) ${}_C\mathbf{r}_D = \overrightarrow{DC}$
$= -2\mathbf{i} - 3\mathbf{j}$

Example 2

Ship A has position vector (20**i** + 10**j**) km. Relative to an observer on A, a second ship, B, has a position vector (3**i** – 8**j**) km. Find the position vector of the second ship.

If A is (20**i** + 10**j**) km from the origin and B is a further (3**i** – 8**j**) km from A it follows that B is (20**i** + 10**j**) + (3**i** – 8**j**) from the origin. Thus ship B has position vector (23**i** + 2**j**) km.

Alternatively the answer could be obtained using the formula ${}_P\mathbf{r}_Q = \mathbf{r}_P - \mathbf{r}_Q$:

We are told that $\mathbf{r}_A = 20\mathbf{i} + 10\mathbf{j}$ and ${}_B\mathbf{r}_A = 3\mathbf{i} - 8\mathbf{j}$.

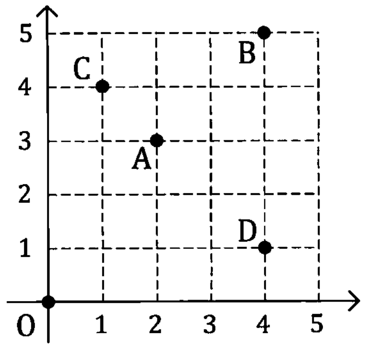
Using
$${}_B\mathbf{r}_A = \mathbf{r}_B - \mathbf{r}_A$$
$$3\mathbf{i} - 8\mathbf{j} = \mathbf{r}_B - (20\mathbf{i} + 10\mathbf{j})$$

Thus
$$\mathbf{r}_B = (3\mathbf{i} - 8\mathbf{j}) + (20\mathbf{i} + 10\mathbf{j})$$
$$= (23\mathbf{i} + 2\mathbf{j})$$

Thus ship B has position vector (23**i** + 2**j**) km, as before.

Exercise 6A

1. The diagram on the right shows the origin O and the points A, B, C and D. Find each of the following in the form **ai** + b**j**.



   (a) The position vector of A.
   (b) The position vector of B.
   (c) The position vector of C.
   (d) The position vector of A relative to B.
   (e) The displacement of A relative to C.
   (f) The position vector of A relative to D.
   (g) The displacement of D relative to A.
   (h) The position vector of B relative to C.
   (i) The position vector of C relative to D.
   (j) The displacement of D relative to C.

2. On a copy of the grid shown on the right show points A to L given the following information.

$\mathbf{r}_A = 2\mathbf{i} + 4\mathbf{j}$ $\quad$ $\mathbf{r}_B = 4\mathbf{i} + 3\mathbf{j}$

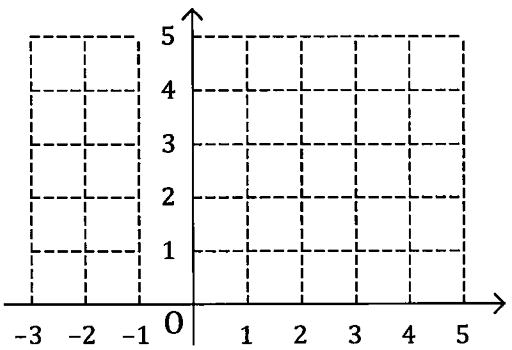
${}_C\mathbf{r}_A = -3\mathbf{j}$ $\quad$ ${}_D\mathbf{r}_A = -\mathbf{i} - \mathbf{j}$

$\mathbf{r}_E = -2\mathbf{i} + 4\mathbf{j}$ $\quad$ ${}_F\mathbf{r}_E = \mathbf{i} + \mathbf{j}$

${}_G\mathbf{r}_D = -4\mathbf{i} - \mathbf{j}$ $\quad$ ${}_H\mathbf{r}_G = \mathbf{i} - \mathbf{j}$

${}_I\mathbf{r}_A = 3\mathbf{i} + \mathbf{j}$ $\quad$ ${}_I\mathbf{r}_J = 3\mathbf{j}$

${}_H\mathbf{r}_K = -5\mathbf{i} + \mathbf{j}$ $\quad$ ${}_I\mathbf{r}_L = 5\mathbf{i} + \mathbf{j}$



3. Ship A has position vector (7**i** + 11**j**) km. Relative to an observer on A a second ship, B, has position vector (−12**i** − 8**j**) km. Find the position vector of ship B.

4. Is $\overrightarrow{AB}$ the displacement of A relative to B or is it the displacement of B relative to A?

5. Ship A has position vector (11**i** − 3**j**) km. Relative to an observer on a second ship B, ship A has position vector (−8**i** − 8**j**) km. How far is ship B from the origin?

6. Ship A has position vector (5**i** + 2**j**) km. Relative to an observer on a second ship B, ship A has position vector (8**i** + 3**j**) km. Find the distance B is from the origin.

7. Point A has position vector 2**i** + 4**j**. Points B and C are such that the position vector of B relative to C is 7**i** − 4**j** and the position vector of C relative to A is −4**i** − **j**. Find the position vector of B.

8. Prove that ${}_D\mathbf{r}_E + {}_E\mathbf{r}_F = {}_D\mathbf{r}_F$.

Relative velocity.

The result ${}_B\mathbf{r}_A = \mathbf{r}_B - \mathbf{r}_A$ can be extended to velocity vectors to determine the velocity of one body relative to another.

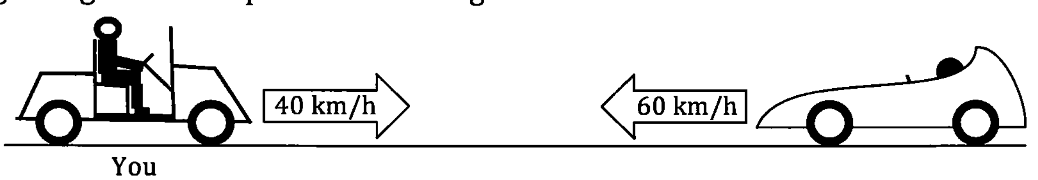
The velocity of A relative to B is given by ${}_A\mathbf{v}_B$ where ${}_A\mathbf{v}_B = \mathbf{v}_A - \mathbf{v}_B$.

By the phrase "the velocity of A relative to B" we mean the velocity of A as seen by an observer on B.

Suppose A and B have velocities $\mathbf{v}_A$ and $\mathbf{v}_B$ respectively. To consider the situation from the point of view of an observer on B we need to imagine a velocity of $-\mathbf{v}_B$ is imposed on the whole system. It will then seem as though B is reduced to rest ($\mathbf{v}_B - \mathbf{v}_B = 0$) and the velocity A has in this system ($= \mathbf{v}_A - \mathbf{v}_B$) will equal the velocity of A as seen by an observer on B. i.e. ${}_A\mathbf{v}_B = \mathbf{v}_A - \mathbf{v}_B$.

Note: All velocities are relative to something. When we write $\mathbf{v}_A$ we usually mean the velocity of A relative to the earth. $\mathbf{v}_A$ could be written ${}_A\mathbf{v}_{Earth}$ and ${}_A\mathbf{v}_B = \mathbf{v}_A - \mathbf{v}_B$ then becomes ${}_A\mathbf{v}_B = {}_A\mathbf{v}_{Earth} - {}_B\mathbf{v}_{Earth}$.

This "velocity of A as seen by an observer on B" was the sort of thing we considered at the beginning of this chapter. Consider again the situation:



If we take $\mathbf{i}$ as a unit vector to the right then:

$$\mathbf{v}_{you} = 40\mathbf{i} \text{ km/h} \quad \text{and} \quad \mathbf{v}_{sports} = -60\mathbf{i} \text{ km/h}.$$

The velocity of the sports car as it appears to "you" is given by ${}_{sports}\mathbf{v}_{you}$ where

$$\begin{aligned} {}_{sports}\mathbf{v}_{you} &= \mathbf{v}_{sports} - \mathbf{v}_{you} \\ &= -60\mathbf{i} - 40\mathbf{i} \\ &= -100\mathbf{i} \end{aligned}$$

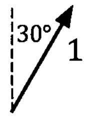
Thus, with the sports car positioned "to the right" of you and moving "left", what you see is the sports car approaching at 100 km/h, as we said when considering this situation intuitively at the beginning of this chapter. This then raises the following question:

Question: We were able to determine the motion of the sports car as seen by "you" before we had met the result ${}_{sports}\mathbf{v}_{you} = \mathbf{v}_{sports} - \mathbf{v}_{you}$ so what is the point of this result?

Answer: The benefit of a result like ${}_A\mathbf{v}_B = \mathbf{v}_A - \mathbf{v}_B$ is that it can be used to find the velocity of one body relative to another body when the directions involved make an intuitive solution difficult. (See example 3).

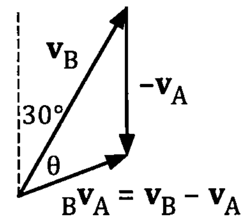
Example 3

Ship A is travelling due North at 10 km/h.
Ship B is travelling on a bearing 030° at 15 km/h.
What is the velocity of ship B relative to ship A? (i.e. What is the velocity of B as seen by an observer on A?)

We are given: $\mathbf{v}_A =$  10 km/h and $\mathbf{v}_B =$ 

15 km/h

and we require: ${}_B\mathbf{v}_A$.



Now
$$ {}_B\mathbf{v}_A = \mathbf{v}_B - \mathbf{v}_A $$
$$ = \mathbf{v}_B + (-\mathbf{v}_A) \quad \text{(See diagram).} $$

By the cosine rule:
$$ |{}_B\mathbf{v}_A|^2 = 10^2 + 15^2 - 2\,(10)\,(15)\cos 30^\circ $$
$$ |{}_B\mathbf{v}_A| \approx 8{\cdot}07 \text{ km/h} $$

By the sine rule:
$$ \frac{|{}_B\mathbf{v}_A|}{\sin 30^\circ} = \frac{|-\mathbf{v}_A|}{\sin\theta} $$
i.e.
$$ \frac{|{}_B\mathbf{v}_A|}{\sin 30^\circ} = \frac{10}{\sin\theta} $$
giving
$$ \theta \approx 38{\cdot}3^\circ $$

Note: The obtuse solution to the above equation, i.e 141·7°, can be ignored in this case because θ is opposite one of the smaller sides of the triangle.

Relative to A, ship B is travelling with speed 8·1 km/hr on a bearing 068°.

Example 4

If $\mathbf{v}_A = 7\mathbf{i} - 2\mathbf{j}$ and $\mathbf{v}_B = 6\mathbf{i} + \mathbf{j}$ find ${}_A\mathbf{v}_B$.

$$ {}_A\mathbf{v}_B = \mathbf{v}_A - \mathbf{v}_B $$
$$ = (7\mathbf{i} - 2\mathbf{j}) - (6\mathbf{i} + \mathbf{j}) $$
$$ = \mathbf{i} - 3\mathbf{j} $$

Thus ${}_A\mathbf{v}_B$, the velocity of A relative to B, is $\mathbf{i} - 3\mathbf{j}$.

Example 5

To a person on a ship travelling with velocity $(15\mathbf{i} + 2\mathbf{j})$ km/h the wind appears to have velocity $(-9\mathbf{i} + 2\mathbf{j})$ km/h. Find the true velocity of the wind.

We are given that $\mathbf{v}_{\text{Ship}} = 15\mathbf{i} + 2\mathbf{j}$ and ${}_{\text{Wind}}\mathbf{v}_{\text{Ship}} = -9\mathbf{i} + 2\mathbf{j}$

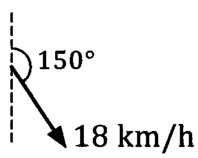
Now $\quad {}_{\text{Wind}}\mathbf{v}_{\text{Ship}} = \mathbf{v}_{\text{Wind}} - \mathbf{v}_{\text{Ship}}$

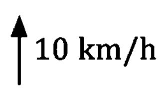
$$\begin{aligned}
\therefore \quad \mathbf{v}_{\text{Wind}} &= {}_{\text{Wind}}\mathbf{v}_{\text{Ship}} + \mathbf{v}_{\text{Ship}} \\
&= (-9\mathbf{i} + 2\mathbf{j}) + (15\mathbf{i} + 2\mathbf{j}) \\
&= 6\mathbf{i} + 4\mathbf{j}
\end{aligned}$$

The true velocity of the wind is $(6\mathbf{i} + 4\mathbf{j})$ km/h.

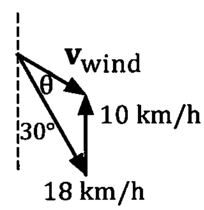
Example 6

To a person on a ship moving at 18 km/h on a bearing 150° the wind appears to come from the South with speed 10 km/h. Find the true velocity of the wind.

We are given: $\mathbf{v}_{\text{ship}} =$ 

${}_{\text{wind}}\mathbf{v}_{\text{ship}} =$ 

and we require: $\mathbf{v}_{\text{wind}}$



Now $\quad {}_{\text{wind}}\mathbf{v}_{\text{ship}} = \mathbf{v}_{\text{wind}} - \mathbf{v}_{\text{ship}}$

$\therefore \quad \mathbf{v}_{\text{wind}} = {}_{\text{wind}}\mathbf{v}_{\text{ship}} + \mathbf{v}_{\text{ship}}$

By the cosine rule $\quad |\mathbf{v}_{\text{wind}}|^2 = 10^2 + 18^2 - 2\,(10)\,(18)\cos 30°$

$\therefore \quad |\mathbf{v}_{\text{wind}}| \approx 10{\cdot}6$ km/h

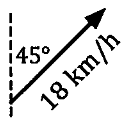
By the sine rule $\quad \dfrac{|\mathbf{v}_{\text{wind}}|}{\sin 30°} = \dfrac{10}{\sin\theta}$ from which $\theta \approx 28{\cdot}2°$

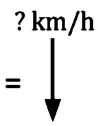
The true velocity of the wind is 10·6 km/h from 302°.

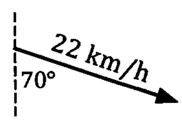
(Again the obtuse solution to the sine rule could be ignored because we chose to determine an angle opposite one of the smaller sides of the triangle.)

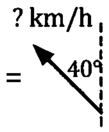
Example 7

To an observer on ship A travelling North-East at 18 km/h the wind appears to come from the North. To an observer on ship B travelling at 22 km/h in direction S70°E the wind appears to come from S40°E. Use a scale drawing to determine the true magnitude and direction of the wind.

We are given: $\mathbf{v}_A =$ 

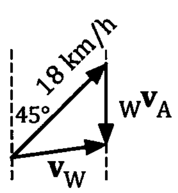
$_W\mathbf{v}_A =$ 

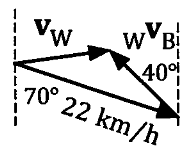
$\mathbf{v}_B =$ 

$_W\mathbf{v}_B =$ 

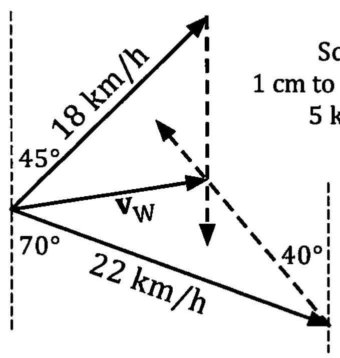
Now $\quad {}_W\mathbf{v}_A = \mathbf{v}_W - \mathbf{v}_A \quad$ and $\quad {}_W\mathbf{v}_B = \mathbf{v}_W - \mathbf{v}_B$

$\therefore \quad \mathbf{v}_W = {}_W\mathbf{v}_A + \mathbf{v}_A \quad$ and $\quad \mathbf{v}_W = {}_W\mathbf{v}_B + \mathbf{v}_B$



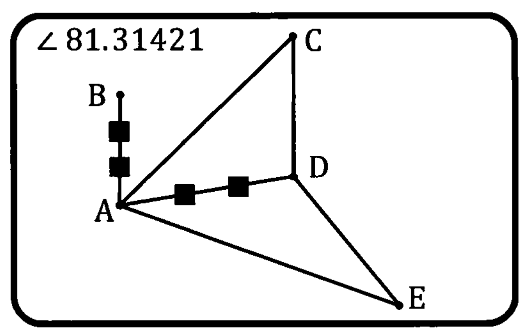


With $\mathbf{v}_W$ common to both triangles a single diagram allows $\mathbf{v}_W$ to be determined:



Scale:
1 cm to represent
5 km/h

or:



The true magnitude and direction of the wind is 13 km/h from S 81° W.

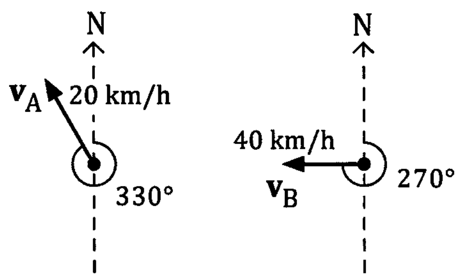
Exercise 6B

In each of the following $\mathbf{v}_A$ is the velocity of A relative to the earth and $\mathbf{v}_B$ is the velocity of B relative to the earth. Find ${}_A\mathbf{v}_B$, the velocity of A relative to B, giving your answers in the form $a\mathbf{i} + b\mathbf{j}$.

1. $\mathbf{v}_A = 2\mathbf{i} - 3\mathbf{j}, \quad \mathbf{v}_B = -4\mathbf{i} + 7\mathbf{j}.$
2. $\mathbf{v}_A = 4\mathbf{i} + 2\mathbf{j}, \quad \mathbf{v}_B = 7\mathbf{i} - \mathbf{j}.$
3. $\mathbf{v}_A = 3\mathbf{i} - 2\mathbf{j}, \quad \mathbf{v}_B = 4\mathbf{i} + 7\mathbf{j}.$
4. $\mathbf{v}_A = 6\mathbf{i} + 2\mathbf{j}, \quad \mathbf{v}_B = 3\mathbf{i} - \mathbf{j}.$

In each of the following $\mathbf{v}_A$ is the velocity of A relative to the earth and $\mathbf{v}_B$ is the velocity of B relative to the earth. Use trigonometry to determine ${}_A\mathbf{v}_B$, the velocity of A relative to B, giving your answer as magnitude and direction.

5.



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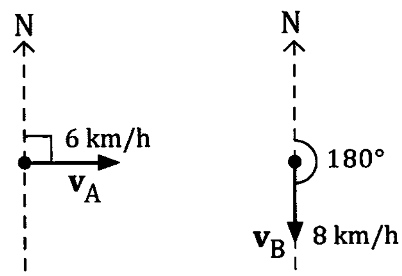

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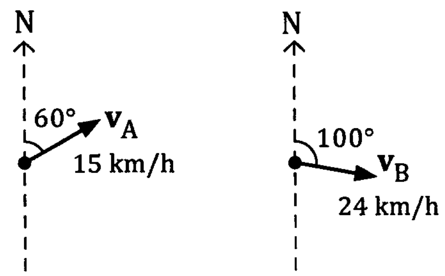

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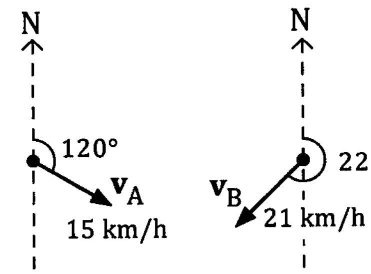
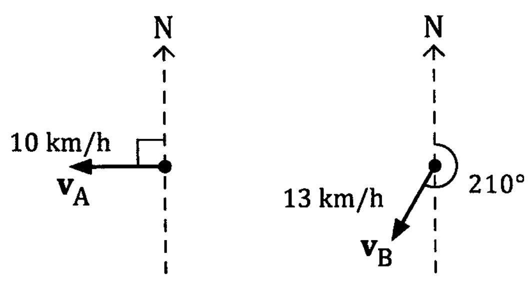


In each of the following $\mathbf{v}_A$ is the velocity of A relative to the earth and $\mathbf{v}_B$ is the velocity of B relative to the earth. Use a scale diagram to determine ${}_A\mathbf{v}_B$, the velocity of A relative to B, giving your answer in magnitude and direction.

9.



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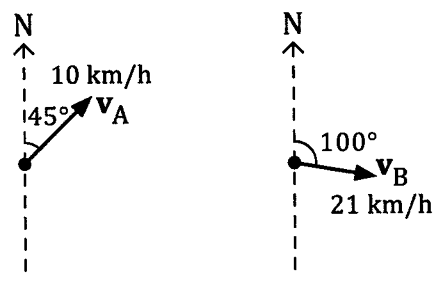


11. Ship A has velocity $(7\mathbf{i} - 10\mathbf{j})$ km/h and ship B has velocity $(2\mathbf{i} + 20\mathbf{j})$ km/h.
Find (a) the velocity of ship A as seen by an observer on B,
(b) the velocity of ship B as seen by an observer on A.

12. Ship A is travelling due West with speed 12 km/h and ship B is travelling on a bearing 060° with speed 15 km/h.
Find (a) the velocity of ship A as seen by an observer on B,
(b) the velocity of ship B as seen by an observer on A.

13. A car is travelling at 100 km/h on a bearing 120° and a motorcycle is travelling due West at 80 km/h. Find the velocity of the motorcyclist relative to the car.

14. If $\mathbf{v}_A = 3\mathbf{i} - 7\mathbf{j}$ and ${}_A\mathbf{v}_B = 2\mathbf{i} + 5\mathbf{j}$ find $\mathbf{v}_B$.

15. If $\mathbf{v}_A = -4\mathbf{i} + 3\mathbf{j}$ and ${}_B\mathbf{v}_A = 2\mathbf{i} + 3\mathbf{j}$ find $\mathbf{v}_B$.

16. Prove that ${}_A\mathbf{v}_B + {}_B\mathbf{v}_C = {}_A\mathbf{v}_C$.

17. If $\mathbf{v}_A$ = 20 km/h due North and ${}_A\mathbf{v}_B$ = 30 km/h due North, find $\mathbf{v}_B$.

18. If $\mathbf{v}_A$ = 80 km/h due South and ${}_A\mathbf{v}_B$ = 60 km/h due South, find $\mathbf{v}_B$.

19. If $\mathbf{v}_A$ = 100 km/h in a direction 060° and ${}_B\mathbf{v}_A$ = 70 km/h in direction 100°, find the magnitude and direction of $\mathbf{v}_B$.

20. To a person on a ship moving with velocity $(7\mathbf{i} + 2\mathbf{j})$ km/h the wind seems to have velocity $(6\mathbf{i} - \mathbf{j})$ km/h. Find the true velocity of the wind.

21. To a person walking with a velocity of $(3\mathbf{i} - 4\mathbf{j})$ km/h the wind seems to have a velocity of $(\mathbf{i} + 2\mathbf{j})$ km/h. Find the true velocity of the wind.

22. To a person on a ship moving South-East at 25 km/h the wind seems to come from the South with speed 15 km/h. Find the true velocity of the wind.

23. To a person walking due North at 5 km/h the wind seems to come from the West with speed 3 km/h. Find the true velocity of the wind.

24. Ship A is travelling due North at 10 km/h. The radar operator on this ship monitors the motion of three other ships B, C and D shown as dots on the radar screen. The movement of the dots give the impression that B is stationary, C is moving South at 3 km/h and D is moving North at 5 km/h but these velocities are all relative to A's motion. Find the true velocities of B, C and D.

25. To a bird flying at 14 km/h on a bearing 160° the wind seems to be from the South at 22 km/h. Find the true velocity of the wind.

26. Ship E is travelling at 15 km/h in direction 040°. The radar operator on this ship monitors the motion of three other ships F, G and H shown as dots on the radar screen. The movement of the dots give the impression that F is moving at 15 km/h in direction 220°, G is moving at 5 km/h due North and H is moving at 30 km/h in direction 300°. However, these velocities are all relative to E's motion. Find the true velocities of F, G and H.

27. The velocity of ship A, as seen by an observer on ship B, is $7\mathbf{i} - 10\mathbf{j}$.
The velocity of ship A, as seen by an observer on ship C, is $13\mathbf{i} - 2\mathbf{j}$.
Find the velocity of ship B as seen by an observer on C.

28. When a cyclist travels East at 20 km/h the wind seems to come from the North-East. When the cyclist travels South at 25 km/h the same wind seems to come from S 30°W. Use a scale diagram to determine the true magnitude and direction of the wind.

29. To a person walking due North at 6 km/h the wind seems to come from N 70°W. To a second person walking due East at 8 km/h the same wind seems to come from S 40°E. Use a scale diagram to determine the true magnitude and direction of the wind.

Miscellaneous Exercise Six.

This miscellaneous exercise may include questions involving the work of this chapter, the work of any previous chapters, and the ideas mentioned in the preliminary work section at the beginning of the book.

1. If $\mathbf{i}$ is a unit vector due East and $\mathbf{j}$ is a unit vector due North express each of the following vectors in the form $a\mathbf{i} + b\mathbf{j}$.

p:	6 units on a bearing 030°.	**q**:	$8\sqrt{2}$ units North-West.
r:	10 units on a bearing 330°.	**s**:	8 units on a bearing S 60°E.

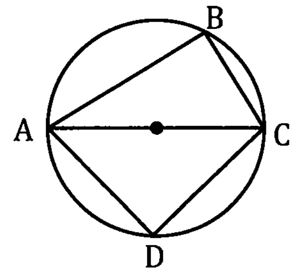
2. Vectors **a**, **b** and **c** are such that $\mathbf{a} = 2\mathbf{i} + 2\mathbf{j}$, $\mathbf{b} = x\mathbf{i} + 5\mathbf{j}$ and $\mathbf{c} = 7\mathbf{i} + y\mathbf{j}$.
If **a** and **b** are parallel and **b** and **c** have equal magnitudes, determine the values of x and y.

3. A school assembly is to be addressed by eight speakers, A, B, C, D, E, F, G and H. How many orders are possible in each of the following situations?
   (a) A must be first.
   (b) A must be first and C must be last.
   (c) A must be first, C must be last and D must immediately follow F.

4. A teacher asked the class to prove the following:

All cyclic quadrilaterals have opposite angles that are supplementary.

One student presented the following proof:



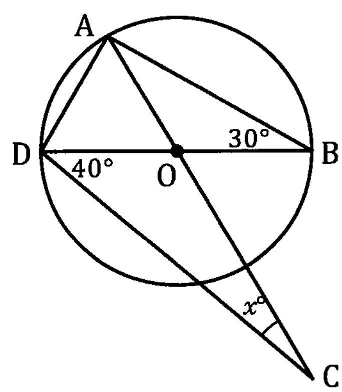
Given:	The cyclic quadrilateral ABCD with AC a diameter of the circle. (See diagram.)			
To prove:	That	$\angle ABC + \angle ADC$	$= 180°$	
	and	$\angle BAD + \angle BCD$	$= 180°$	
Proof:		$\angle ABC$	$= 90°$	(Angle in a semi-circle.)
		$\angle ADC$	$= 90°$	(Angle in a semi-circle.)
	$\therefore$	$\angle ABC + \angle ADC$	$= 180°$, as required.	⟵ ①
		$\angle ABC + \angle ADC + \angle BAD + \angle BCD$	$= 360°$	(Quadrilateral angle sum.)
	Thus	$180° + \angle BAD + \angle BCD$	$= 360°$	(Using ①.)
	$\therefore$	$\angle BAD + \angle BCD$	$= 180°$, as required.	

If the answer were to be marked out of ten how many marks would you award the above response and why?

5. In the diagram on the right points A, B and D lie on the circle centre O with BD as a diameter.

C is a point outside of the circle with A, O and C collinear and $\angle ACD = x°$.

Given that	$\angle ABD$	=	30°
and	$\angle BDC$	=	40°
prove that	x	=	20.



6. Point P has position vector $-7\mathbf{i} + 13\mathbf{j}$ and point Q has position vector $13\mathbf{i} - 2\mathbf{j}$. Point R lies on the straight line joining P to Q and is such that

$$PR : PQ = 3 : 5.$$

Find
(a) how far R is from Q,
(b) the position vector of R,
(c) how far R is from the origin.

7. If a statement "if P then Q" is true then which of the following must also be true:

    If Q then P.
    If not P then not Q.
    If not Q then not P.

8. How many different combinations of five letters, all different, can be formed from the letters of the alphabet if each combination must contain two vowels and three consonants?
   How many of these combinations contain an a or an e?
   (Remember in mathematics we interpret "or" to mean "one or the other or both", i.e we interpret it to mean "at least one of".)

9. Five places exist on a course for teachers. The five are to be chosen from the 11 teachers who have expressed interest in attending. These 11 come from three education districts with 6 from District A, 4 from District B and 1 from District C.
   How many different groups of five are there?
   How many different groups of five are there if each group must contain at least one teacher from each of the three districts?

10. Three soccer team managers are to be invited to attend a TV talk program.
    Eight managers are shortlisted: A, B, C, D, E, F, G, H.
    (a) How many different combinations of three managers are possible?
    (b) Bitter rivalry exists between managers C and D so three options are considered:

    Option I: Invite both C and D as lively debate should ensue.

    Option II: Avoid conflict and, to be fair, invite neither C nor D.

    Option III: Avoid the "invite both C and D" situation but allow any other group of 3 that is decided upon.

    For each of these three options how many different combinations of three managers are possible?

A B Ɔ D E F G H